

**ÇANKAYA UNIVERSITY**  
Department of Mathematics and Computer Science

**MATH 325**  
**Introduction to Abstract Algebra I**

Practice Final Exam

January 8, 2007

17:40-19:30

- This exam is meant to suggest the kind, difficulty, and number of problems on the real exam. Note that topics covered in the course, but not included on this practice exam, still might appear on the real exam.
- **Repeat:** Topics covered in the course, but not included on this practice exam, still **might appear on the real exam.**
- Set aside 110 min. during which you can work on this exam without interruption. Refer to no books, notes, or calculators while working on the exam. This will give you a sense of your level of preparation for the real exam.

1. Is  $\mathbb{Z}_3 \oplus \mathbb{Z}_9$  isomorphic to  $\mathbb{Z}_{27}$ ? Why?
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**2.** Determine the number of elements of order 15 and the number of cyclic subgroups of order 15 in  $\mathbb{Z}_{30} \oplus \mathbb{Z}_{20}$ .

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3. Prove that a factor group of a cyclic group is cyclic.
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4. What is the order of the element  $14 + \langle 8 \rangle$  in the factor group  $\mathbb{Z}_{24} / \langle 8 \rangle$ ?

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5. Determine the order of  $(\mathbb{Z} \oplus \mathbb{Z}) / \langle (2, 2) \rangle$ . Is the group cyclic?

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6. Let  $|G| = 8$ . Show that  $G$  must have an element of order 2.

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7. Suppose that  $K$  is a proper subgroup of  $H$  and  $H$  is a proper subgroup of  $G$ . If  $|K| = 42$   $|G| = 420$ , what are the possible orders of  $H$ ?

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8. Suppose that  $a$  has order 15. Find all left cosets of  $\langle a^5 \rangle$  in  $\langle a \rangle$ .

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